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Well-Posed Integro-Differential Equations in a New Pair of Weight-Free Sobolev Spaces

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Abstract—We are dealing with general boundary-value problem for linear integro-differential equations on a segment of the real axis. In the case under consideration the order of internal differential operators is higher than the order of exterior one. We prove that the problem is well-posed in the Hadamard sense in a new pair of weight-free Sobolev spaces.

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Introduction. Let $r, m+1 \in \mathbb{N}$ be fixed numbers such that $r > m \geq 1$. We consider general boundary-value problem

$$R_i(x) = 0, \quad i = \overline{1, m}, \tag{1}$$

for integro-differential equation

$$Kx \equiv x^{(m)}(t) + \sum_{k=1}^{m} g_k(t) x^{(m-k)}(t) + \sum_{j=0}^{r} \int_{-1}^{+1} h_j(t,s) x^{(j)}(s)\, ds = y(t), \quad -1 \leq t \leq 1, \tag{2}$$

where $\{R_i\}$ are linearly independent functionals on the space of $(m-1)$ times continuously differentiable on segment $[-1, 1]$ functions, and functions $y(t)$, $g_k(t)$, $k = \overline{1, m}$, $h_j(t, s)$, $j = \overline{0, r}$, are given.

In the paper [1] this problem is referred to conditionally well-posed, because under certain restrictions on smoothness of coefficients it is well-posed in the Hadamard sense for suitable pairs of spaces X of desired elements and Y of right-hand sides. These pairs are built now for Cauchy problems for Eq. (2) (see, e.g., [2]). The corresponding spaces are the Sobolev ones with special weights depending on difference of orders of internal and exterior differential operators. Unfortunately, the technique of approximations in these spaces is not developed in sufficient degree. Therefore, the problem of building of convenient pairs (X, Y) keeps its urgency.

In this paper we offer new family of pairs of these spaces. We consider without loss of generality that functionals R_i are discrete and have the form

$$R_i(x) = x^{(i-1)}(-1), \quad i = \overline{1, m}.$$

This fact can be proved by introducing new desired function

$$z(t) = x(t) + \sum_{i=1}^{m} \left[R_i(x) - x^{(i-1)}(-1) \right] \cdot \frac{(t+1)^{i-1}}{(i-1)!}.$$

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